Differentiating and equating to zero,

$$\frac{du}{dx}=\pm\frac{R-x}{t\sqrt{[2Rx-x^2+r^2]}}-1/t=0.$$

Reducing, $2x^2-4Rx=r^2-R^2$.

Whence $x=R\pm\sqrt{\left(\frac{R^2+r^2}{2}\right)}$.

Making $r=\frac{1}{3}R$, and taking the minus sign, $x=R(1-\sqrt{\frac{5}{9}})$.

$x=.25464R$, or a little more than one-fourth of the radius.

II. Solution by HENRY HEATON, M. Sc., Atlantic, Ia.

To secure the greatest result, the area occupied by the spring when wound or unwound must be one-half that between the hub and the inner circumference of the barrel. This area is $\frac{4}{9}r^2\pi$, and the area occupied by the hub and spring when the latter is wound, is $\frac{5}{9}r^2\pi$. Hence the radius of the circumference lying within the annulus is $r'=\frac{1}{3}r\sqrt{5}=.745r$.

$\therefore r-r'=.254r$.

III. Solution by P. H. PHILBRICK, C. E., Lake Charles, La.

Let a=area of the cross-section of the barrel; then $(\frac{1}{3})^2a=\frac{1}{9}a$=the area of the cross-section of the hub; $\frac{8}{9}a$=area around the hub, and $\frac{4}{9}a$=one-half of that area; $\frac{5}{9}a$=area of cross-section of hub and spring.

Hence both hub and spring occupy $\sqrt{\frac{5}{9}}=.7454$ of the radius of the barrel, and the unwound spring occupies $1-.7454=.2546$ of that radius.

MECHANICS.

81. Proposed by JAMES S. STEVENS, Professor of Physics, The University of Maine, Orono, Me.

Two iron spheres whose weights are a and b, and a is greater than b, are suspended over a frictionless pulley so that they move in a liquid medium of density δ. Assume that the density of the iron is δ', what would be the spaces passed over (downward by a and upward by b) in the first four seconds, if the spheres start from rest?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let m=the mass$=(a-b)/g$......(1).

v=the velocity at time t, Rv^2=resistance. The resistance is the sum of the resistances for both spheres.

Let A=the greatest cross-section of $a=\dfrac{\sqrt[3]{[36a^2\pi g\delta'}}{4g\delta'}$.

Let B=the greatest cross-section of $b=\dfrac{\sqrt[3]{[36b^2\pi g\delta'}}{4g\delta'}$.

From Rankine, $Rv^2=\frac{kev^2}{2g}(A+B)$ where $k=0.51$ for the sphere, e=the weight of unit of fluid$=g\delta$.

$$\therefore R=\frac{0.51\delta\sqrt[3]{[36\pi g\delta']}[\sqrt[3]{(a^2)}+\sqrt[3]{(b^2)}]}{8g\delta'}\ldots\ldots(2).$$

Equation of motion is, $m(dv/dt)=mg-Rv^2$.

$$\therefore t=\frac{m}{2\sqrt{(gmR)}}\log_e\left(\frac{\sqrt{(mg)}+v\sqrt{R}}{\sqrt{(mg)}-v\sqrt{R}}\right).\quad \text{Let } n=\sqrt{(gR/m)}\ldots\ldots(3).$$

$$\therefore v=(dx/dt)=\frac{mn}{R}\left(\frac{e^{nt}-e^{-nt}}{e^{nt}+e^{-nt}}\right).\quad \therefore x=\frac{mn}{R}\int_0^4\left(\frac{e^{nt}-e^{-nt}}{e^{nt}+e^{-nt}}\right)dt.$$

$$\therefore x=\frac{m}{R}\log_e\left(\frac{e^{4n}+e^{-4n}}{2}\right)\ldots\ldots(4).$$

(1), (2), (3) in (4) gives the result required.
Let $g=32.16$, $\delta=1$, $\delta'=7.8$, $a=40$ lbs., $b=7.84$ lbs.
$\therefore m=1$, $R=.1212$, $n=1.974$.
$\therefore x=8\frac{1}{3}\log_e\left(\frac{e^{7.896}+e^{-7.896}}{2}\right)=8\frac{1}{3}\log_e(1343.3184)$.
$\therefore x=60.0233$ feet.

II. Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

Let T be the tension of the string.

The mass of the liquid displaced by the heavier body is ad/gd', of that displaced by the lighter body is bd/gd'; measuring distances downward for the heavier body and upward for the lighter body, assuming that resistance of the liquid to motion varies as the square of the velocity, the equations of motion for the heavier and the lighter body are respectively:

$$a\frac{d^2s}{dt^2}=ag-\frac{ad}{d'}-kv^2-gT\ldots\ldots(1).$$

$$b\frac{d^2s}{dt^2}=-bg+\frac{bd}{d'}-kv^2+gT\ldots\ldots(2).$$

Eliminating T, $\frac{d^2s}{dt^2}=\frac{(a-b)(d'g-d)}{(a+b)d'}-2kv^2\ldots\ldots(3).$

By integration twice:

$$s=\frac{(a+b)d'}{(a-b)(d'g-d)}\log\left[\frac{l\,t^{\sqrt{[2k(a-b)(d'g-d)]}/\sqrt{[(a+b)d']}}+l^{-t\sqrt{[2k(a-b)(d'g-d)]}/\sqrt{[(a+b)d']}}}{2}\right]$$

$$\text{or } s=\frac{(a+b)d'}{(a-b)(d'g-d)}\log(\cosh t\sqrt{[2k(a-b)(d'g-d)]}/\sqrt{[(a+b)d']}).$$

$$s=\frac{1}{p^2}\log(\cosh pt) \text{ where } p=\sqrt{\frac{2k(a-b)(d'g-d)}{(a+b)d'}}.$$

For four seconds, $s=\frac{1}{p^2}\log(\cosh 4p)$.

[See Bowser's *Analytic Mechanics*, page 314, ex. 5, where $v=0$ and $d=0$, of equation (3) above.]

Also solved by *ELMER SCHUYLER.*

AVERAGE AND PROBABILITY.

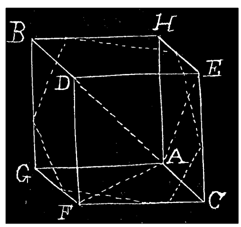
61. Proposed by COL. CLARKE.

A cube being cut at random by a plane, what is the chance that the section is a hexagon? [Erom Williamson's *Integeal Calculus.*]

Solution by LEWIS NEIKIRK, Graduate Student, University of Colorado, Boulder, Col.

I. Preliminary Investigation.

Let S, the random section, be determined by the coördinates p, φ, and θ, θ being the angle between p and its projection on $ACFG$ and φ the angle between AC and the projection of p. Let P be the point of intersection of p and S. Also let p increase from zero for φ and $\theta<\frac{1}{2}\pi$. S, starting with three sides at A, gains three more, one at each of the corners C, G, and H; and loses three, one at each of the corners B, E, and F. φ and θ determine the order in which these gains and losses shall occur, and plainly S can be hexagonal only when the first loss is antedated by all three gains.



For p in the diagonal AD ($\varphi=\frac{1}{4}\pi$, $\theta=\cot^{-1}\sqrt{2}$), the three gains are simultaneous and are followed by three simultaneous losses. For p as an element of the area DAF ($\varphi=\frac{1}{4}\pi$, $\theta<\cot^{-1}\sqrt{2}$), one gain at H is followed by two more at C and G; then two losses at B and E, followed by one loss at F. For p as an element of the areas DAE and DAB ($\varphi<\frac{1}{4}\pi$ and $\theta=\tan^{-1}\cos\varphi$, and $\varphi>\frac{1}{4}\pi$ and $\theta=\tan^{-1}\sin\varphi$) there is a like sequence of gains and losses. So far it has been easy to enumerate the exact order in which all the gains and losses occur.

For p within the solid angle $A—DECF$ ($\varphi<\frac{1}{4}\pi$ and $\theta<\tan^{-1}\cos\varphi$) the first loss occurs at B, and the last gain at C; the order and place of the remaining gains and losses would be difficult to enumerate and is in any event immaterial